

# Contents

<b>Introduction</b>	<b>1</b>
<b>1. Colour ordering and primitive amplitudes</b>	<b>5</b>
1.1. Tree-level amplitudes in pure gauge theory . . . . .	5
1.2. Quark–gluon tree-level amplitudes . . . . .	9
1.3. Colour decomposition and primitive amplitudes at one-loop order . . .	13
1.4. Colour summed squared helicity amplitudes . . . . .	18
<b>2. Integrand reduction techniques at one-loop order</b>	<b>21</b>
2.1. The general structure of one-loop amplitudes . . . . .	21
2.1.1. Kinematical conventions . . . . .	21
2.1.2. Scalar integral basis . . . . .	23
2.1.3. Comments on the rational part . . . . .	24
2.1.4. Scheme dependence and universal pole structure . . . . .	25
2.2. The van Neerven-Vermaseren basis . . . . .	27
2.2.1. Construction of the basis vectors in four space-time dimensions .	29
2.2.2. Extension of the basis vectors to five space-time dimensions . . .	32
2.2.3. The loop momentum in terms of the van Neerven-Vermaseren basis . . . . .	33
2.3. The general structure of the one-loop integrand . . . . .	34
2.4. Rational part versus cut-constructible part and the choice of master in- tegrals . . . . .	41
2.5. Computation of the cut-constructible part . . . . .	55
2.5.1. The box coefficients . . . . .	55
2.5.2. The triangle coefficients . . . . .	57
2.5.3. The bubble coefficients . . . . .	63
2.6. Computation of the rational part . . . . .	66
2.6.1. The rational pentagon contribution . . . . .	67
2.6.2. The rational box contribution . . . . .	68
2.6.3. The rational triangle contribution . . . . .	72
2.6.4. The rational bubble contribution . . . . .	73
2.7. Tree-level amplitudes for the unitarity cuts . . . . .	75
<b>3. Numerical computation of one-loop QCD amplitudes</b>	<b>81</b>
3.1. Validation and numerical accuracy . . . . .	81
3.2. Runtime performance . . . . .	93

<b>4. Four-jet production at next-to-leading order for the Large Hadron Collider</b>	<b>99</b>
4.1. General outline of the computation . . . . .	99
4.2. Numerical setup . . . . .	101
4.3. Results . . . . .	102
<b>Conclusion</b>	<b>108</b>
<b>Appendix</b>	<b>111</b>
<b>A. Spinor-helicity methods</b>	<b>113</b>
<b>B. Algorithmic realisation of colour ordered off-shell recursion</b>	<b>119</b>
B.1. Constructive implementation of the Berends-Giele recursion . . . . .	119
B.2. Extension of the Berends-Giele recursion to arbitrary quark lines . . . . .	122
<b>C. Some Gram determinant relations</b>	<b>127</b>
<b>D. Alternative methods to disentangle the integral coefficients</b>	<b>129</b>
D.1. The cut-constructible integral coefficient $b^{(0)}$ . . . . .	129
D.2. The rational integral coefficient $b^{(9)}$ . . . . .	130