Abstract

Regardless of the field, measurements are essential for validating theories and making well-founded decisions. A criterion for the validity and comparability of measured values is their uncertainty. The "Guide to the Expression of Uncertainty in Measurement" (GUM) provides a standardized framework for determining and interpreting measurement uncertainty. Still, in room acoustical measurements, the application of these rules is not yet widespread. Firstly, this is due to the fact that well established 2-CH-FFT correlation techniques rely on a complex principle, which is not covered by the classical guide. In addition, the effect of input variables on an individual measurement can only be determined after considerable effort. An example are fluctuations of room acoustical quantities over small distances between measurement locations in concert halls. This variation of the sound field by position is sometimes considerable and can only be predicted in relatively simple boundary value problems. This raises the question of the validity and interpretability of room acoustical measurements.

The goal of this thesis is to provide a GUM-compliant discussion of uncertainties in measuring room acoustical single-number quantities. This starts with a structured search of variables that potentially influence the measurement of room impulse responses. In a second step, this uncertainty is propagated through the algorithm that determines room acoustical single-number quantities.

Further emphasis is placed on the investigation of spatial fluctuations of the sound field in auditoria. The influence of an uncertain measurement position on the overall measurement uncertainty is discussed. To reach general conclusions, the relation between changes in the measurement location and the corresponding changes in measured room acoustical quantities is investigated empirically in extensive measurement series. To this end, a measurement apparatus was designed that allows automatic, high-resolution sampling of sound fields over large areas. The collected data creates the foundation to apply the principle of uncertainty propagation using a Monte Carlo method.

This study shows how precisely a measurement position must be defined to ensure a given uncertainty of room acoustical single-number quantities. The presented methods form a foundation that can be flexibly extended in future investigations to include additional influences on the measurement uncertainty.

Kurzfassung

Unabhängig vom Fachgebiet sind Messungen essentiell für die Validierung von Theorien und für das Treffen fundierter Entscheidungen. Merkmal für die Aussagekraft und Vergleichbarkeit von Messwerten ist unter Anderem deren Unsicherheit. Für die Bestimmung und die Interpretation der Messunsicherheit stellt der *Guide to the Expression of Uncertainty in Measurement* (GUM) einen standardisierten Rahmen bereit. Bei raumakustischen Messungen ist die Anwendung dieses Regelwerks bisher noch nicht grundsätzlich verbreitet. Das liegt einerseits daran, dass mit der weit verbreiteten Korrelationsmesstechnik ein komplexes Messprinzip verwendet wird, das im klassischen Leitfaden nicht behandelt wird. Außerdem ist die Wirkung von Eingangsgrößen, die eine Messung beeinflussen können, nur mit größtem Aufwand im Einzelfall bestimmbar. Beispiel dafür sind Fluktuationen raumakustischer Kenngrößen über kleinste Abstände zwischen Messorten in Konzertsälen. Diese zum Teil beachtliche Änderung des Schallfeldes über den Ort wirft die Frage nach der Aussagekraft und Interpretierbarkeit raumakustischer Messungen auf.

Ziel dieser Arbeit ist eine GUM konforme Diskussion der Unsicherheit beim Messen raumakustischer Einzahlkennwerte. Begonnen wird dabei mit einer strukturierten Suche der Größen, die die Messung von Raumimpulsantworten beeinflussen könnten. In einem zweiten Schritt wird diese Unsicherheit durch den Algorithmus zur Bestimmung raumakustischer Einzahlkennwerte propagiert.

Ein weiterer Schwerpunkt wird auf die Untersuchung von räumlichen Änderungen des Schallfeldes in Auditorien gelegt. Es wird der Einfluss eines unsicher bestimmten Messorts auf die Messunsicherheit diskutiert. Um möglichst allgemeingültige Aussagen treffen zu können wird der Zusammenhang zwischen einer Änderung des Messortes und der korrespondierenden Änderung raumakustischer Kenngrößen in umfangreichen Messreihen empirisch untersucht. Dazu wurde ein Messapparat gebaut mit dem Schallfelder hochauflösend und vollautomatisch über große Flächen abgetastet werden können. Die so gesammelten Daten bilden die Grundlage für die Berechnung der Unsicherheitsfortpflanzung mit einer Monte Carlo Methode.

Die Ergebnisse dieser Untersuchung zeigen, wie genau ein Messort bei raumakustischen Messungen definiert werden muss, um eine zuvor festgelegte Unsicherheit raumakustischer Einzahlkennwerte zu gewährleisten. Die vorgestellten Methoden bilden eine Grundlage, die flexibel erweitert werden kann, um weitere Einflüsse auf die Messunsicherheit in zukünftigen Untersuchungen zu berücksichtigen.

1 Introduction

In architecture, concert halls, lecture rooms or open plan offices are designed for different types of communication depending on their specific intended uses. In rooms for speech, it is important that the speaker's loudness is sufficient, but also that individual syllables are clearly intelligible. For music, there are additional aspects such as spatial sound or spectral balance that need to be considered. In some environment, such as offices, distracting communication is unwanted and should be minimized. The acoustician supports the architect in the design to make sure critical limits for good communication are safely met and the room is acoustically suitable for its purpose.

Acoustics is an interdisciplinary field and employs tools and methods from a wide range of other specialized areas. In architectural acoustics, the impulse response has proven to be a very useful concept from system theory. The impulse response, which describes the transmission of information from a source to a receiver also holds in auditorium acoustics since the room in which the sound source and the listener are located can be understood as a transmission channel. The room's response to an impulse is the direct sound that travels from the source to the listener, the sound reflections from the surfaces and the lingering reverberation. Since there is a physical relation between the room's geometry and the sound transmission, it is intuitively evident why the impulse response is so useful for the acoustic design of auditoria. Experts can interpret impulse responses and easily recognize how syllables of a speaker are supported by early reflections or see how a long reverberation blurs successive syllables in time and thus impairs communication.

Room impulse responses can be measured with special equipment and contribute to the acoustic planning process by providing the data to place future design decisions on solid ground and quantify the effectiveness of previous design decisions. In general, measurements are of core importance in science and practice when it comes to proving theories or making well-founded decisions. The suitability of measurements as the basis for a valid argument depends a lot on the data's associated uncertainties. Modern measurement methods (ISO 18233,

2006) to measure transfer functions and their associated impulse responses (IR) using maximum length sequences or swept sine signals are common tools in all areas of acoustics (Müller & Massarani, 2001). In architectural acoustics, room impulse responses are regularly analyzed to determine single-number quantities that serve as predictors for sound perception (see Section 2.1.2). Clarity C_{80}, for example, scales the perceived distinctness of a sound in time from highly detailed ($\approx 7\,\mathrm{dB}$) to blurred ($\approx -5\,\mathrm{dB}$). Provided that the measured environment features the properties of linear time invariant (LTI) systems and that a sufficient signal-to-noise ratio is achieved, acoustical measurements of impulse responses and room acoustic quantities are usually considered to be rather accurate.

This perspective was briefly challenged in a reflex reaction to findings of de Vries, Hulsebos, and Baan (2001). Under quasi-repeatability conditions at Concertgebouw Amsterdam, RIRs were measured every 5 cm along a line following a row of seating. The data de Vries and his team collected shows how room acoustic single number quantities fluctuate over the surveyed distance; Figure 1.1 illustrates this for C_{80} (ISO 3382-1, 2009). In facetious discussions, auditoria were compared with random number generators and the question of explanatory power in room acoustics measurements was raised.

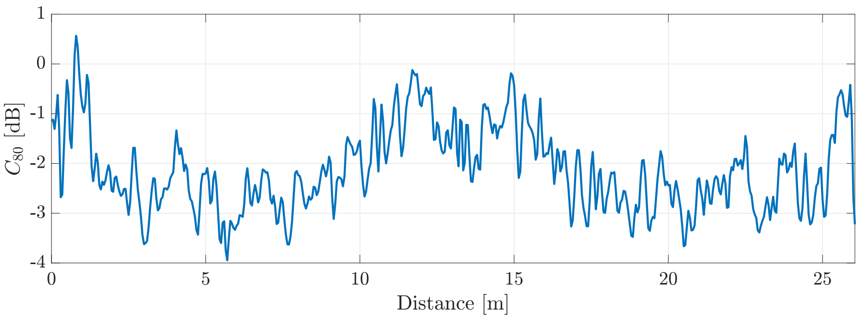

Figure 1.1: Spatial distribution of clarity C_{80} at the 1 kHz octave band along a line measured at a concert hall.

Of course, this fabricated perspective does not appreciate the deterministic character of sound propagation adequately, but the reference to a reproduction problem in measurements is well-founded: If room acoustic quantities change over such small distances, how can measurements be reproduced at another time? How can the acoustic effectiveness of a modification to a building be verified when the expected acoustic change is obscured by strong fluctuations?

At first glance it may seem that de Vries et al. (2001) merely confirm findings from earlier investigations that were conducted in the advent of the ISO 3382 (1975) revision (e.g., J. S. Bradley and Halliwell (1988); Hidaka, Beranek, and Okano (1995); Pelorson, Vian, and Polack (1992)). After all, these studies also discuss strong variations in room acoustic parameters with relatively small spatial displacements of sources or receivers. The key difference is in the way the collected data was analyzed and interpreted. Prior to de Vries, the determined spread in room acoustic parameters was discussed statistically, such that an adequately large sample size would be sufficient to correctly determine the variance in a statistical population. Consequently, these findings lead to the requirement to measure at numerous positions distributed throughout the auditorium and, hence, provide a sufficiently large sample size to calculate average values (ISO 3382, 1997).

But with growing experience in using the revised standard it soon became more and more evident that the underlying cause-and-effect chain was not fully factored in: J. S. Bradley (1994) demonstrated that calculating hall-spanning parameter averages comes with the potential to flatten out characteristic patterns. This may lead to a point where auditoria, fundamentally different in shape, are no longer distinguishable in their summary statistics. Today, there is a common understanding that averaging over all measurement positions to gain a hall mean value seems (except for the reverberation time) generally unhelpful (Barron, 2005; J. S. Bradley, 2005). This interpretation is justified within the large-scale dimensions of an auditorium, but it does not recognize the parameter variations encountered within smaller distances. Follow-up investigations by Nielsen, Halstead, and Marshall (1998), Sekiguchi and Hanyu (1998) and Okano, Beranek, and Hidaka (1998) indicate that the phenomenon continued to be a target of interest.

In this course of development the initially quoted study by de Vries et al. (2001) marks an important milestone as it provides high-resolution data that shows how the acoustic quantities fluctuate over a wide range of distances, starting from a few cm to the dimensions of a concert hall. This can be interpreted as metrological evidence towards an influence factor of measurement position that seemed "downgraded" by averaging over a number of locations. Taking the *sampling position* as a relevant input quantity it becomes possible to refine the statistical discussion and investigate how this contribution influences the result of acoustical measurements. This approach can create a context for how uncertain room acoustics measurements are and identify the subtleties worth interpreting.

The standardized tools for this discussion are provided by the "Guide to the expression of uncertainty in measurement" (GUM, ISO Guide 98-3 (2008)) that places the original principles of Gaussian *error propagation* on a wider foundation. In the first step a relationship needs to be established that quantifies how a change in measurement position yields a change in the measured result. This marks an advance to the previous statistical discussion as the sampling position is recognized as an influence quantity. It also moves the discussion away from insulated individual cases to a general discussion of a broad spectrum of sound fields and thus helps to assess how significant the "validity" problem generally is. In light of the raised question of reproducibility in room acoustic measurements, the measurement function marks the foundation to investigate a derived question of practical relevance: How precisely need measurement positions (source and receiver) be defined?

1.1 Defining the scope of this work

Against this background, it is important to discuss the uncertainty of room acoustic measurements. In preparation to determine the measurement model (ISO Guide 98-3, 2008, 4.1.1) empirically, the following questions must be discussed.

- What is the uncertainty of room acoustic impulse response measurements?
- What is the uncertainty of room acoustic ISO 3382-1 (2009) quantities?

In preparing the measurement model this question needs to be answered:

- How do room acoustic quantities change when the measurement position is changed by a given distance?

Based on the answer to the previous question, this problem of practical relevance should be addressed:

- How accurately need measurement positions be defined?

2

Fundamentals and previous work

Discussing these research questions requires a set of tools that will be briefly outlined in this chapter. An initial focus is placed on the theoretical groundwork, as these basics identify the driving forces and the relevant variables; in the next step the measurement uncertainty will be of interest. In this regard it is reasonable to first examine the status quo in room acoustical measurements and then gradually traverse to the framework that permits a uniform discussion of uncertainties in measurements.

2.1 Theoretical principles behind spatial fluctuations

2.1.1 Amplitude distribution due to changes in position

Initial work on spatial fluctuations of the sound field in rooms can be traced back to Kuttruff and Thiele (1954) and Kuttruff (1954). While this work was originally focused on the frequency dependency of the sound pressure in rooms, as part of the investigations that led to what is now known as the "Schroeder Frequency" (Schröder, 1954), the initial empirical study by Kuttruff and Thiele (1954) also discussed the the sound pressure's spatial dependency.

The starting point is the sound field in a rectangular room with the dimensions L_x, L_y, L_z and rigid surfaces. At characteristic eigenfrequencies ω_i for $\forall n \in \mathbb{N}_0$ (here Equation 2.1.1, from Kuttruff (2000), Eq. 3.15) the cartesian components of the respective wave vector k_i meet the scenario's boundary conditions (particle velocity $v_i = 0$ at the room's surfaces) so that the system can oscillate.

$$\omega_i = c\pi\sqrt{\left(\frac{n_{x,i}}{L_x}\right)^2 + \left(\frac{n_{y,i}}{L_y}\right)^2 + \left(\frac{n_{z,i}}{L_z}\right)^2} = ck_{n_x n_y n_z} \tag{2.1.1}$$

At each of the system's ω_i's, standing waves develop. For a single mode the sound pressure at a position $\mathbf{r}_r = (x_r, y_r, z_r)^T$ can be determined through Equation 2.1.2 (Kuttruff, 2000, Eq. 3.16).

$$p_i(\mathbf{r}) = \cos\left(\frac{n_{x,i}\pi x_r}{L_x}\right)\cos\left(\frac{n_{y,i}\pi y_r}{L_y}\right)\cos\left(\frac{n_{z,i}\pi z_r}{L_z}\right) \tag{2.1.2}$$

Based on the principle of reciprocity (Morse & Ingard, 1968, p.134) it can be understood that this relation is valid for the source and the receiver alike. The spatial cosine relationship includes the characteristic nodal points for a standing wave at a given frequency when the cosine's argument is $(2n+1)\pi/2$. At these nodes the sound pressure is naturally zero. Reciprocally, at the same positions the sound field cannot be excited at that frequency. The full relationship is shown in Equation 2.1.3 (Kuttruff, 2000, Eq. 3.10) with the numerator featuring the mathematical representation of the standing wave excited and sampled at the positions $\mathbf{r}_s$ and $\mathbf{r}_r$. This term, thus, shows a dependency on the location and is hence responsible for the spatial fluctuation of the sound field.

$$p(\omega, \mathbf{r}_r) = jQc^2\omega\rho_0 \sum_i \frac{p_i(\mathbf{r}_s)p_i(\mathbf{r}_r)}{(\omega^2 - \omega_i^2 - 2j\delta_i\omega_i - \delta_i^2)K_i} \tag{2.1.3}$$

Obviously, a realistic system requires some damping to balance the source's energy influx and so become stable. This is recognized in the denominator where the ideal dirac-delta-like eigenfrequencies are expanded through the damping constant δ_i to Cauchy-Lorentz functions with characteristic quality factors.

The factors in front of the sum in Equation 2.1.3 recognize the physical properties of the point source. The volume velocity Q includes the harmonic oscillation $e^{j\omega t}$. K_i is a normalization constant for the standing waves (Kuttruff, 2000, Eq. 3.3),

$$K_i = \iiint_V p_i^2(\mathbf{r})dV. \tag{2.1.4}$$

The sigma sign in Equation 2.1.3 indicates that at a given position $\mathbf{r}_s$ and $\mathbf{r}_r$ more than one mode is excited and so the emerging sound pressure level is a result of a sum over i eigenfrequencies. Schröder (1954) argues that when the number of eigenmodes is sufficiently large the Lindeberg-Lévy central limit theorem will take effect and the summary distribution of the real and imaginary parts of the sound pressure will become normally distributed. From there it is only the small step of taking the absolute value of the complex sound pressure to arrive at the Rayleigh distributed (linear) sound pressure amplitude p with a probability density function f_p given in Equation 2.1.5 (Johnson, Kotz, & Balakrishnan, 1994; Rayleigh, 1880). σ_p^2 refers to the variance of the sound pressure's real or imaginary parts $\mathrm{Re}(p)$, $\mathrm{Im}(p)$.

$$f_P(p) = \frac{p}{\sigma_p^2} e^{-\frac{p^2}{2\sigma_p^2}} \tag{2.1.5}$$

The probability density function of the sound pressure level L_p can be determined to be

$$f_{L_p} = \frac{\ln(10)}{2} \frac{p_0^2 10^{L_p/10}}{\sigma_p^2} e^{-\frac{p_0^2 10^{L_p/10}}{2\sigma_p^2}} \tag{2.1.6}$$

by using the *transformation theorem for probability densities* (Johnson et al., 1994, pp. 14-15, Eq. 12.32).

Having considered the central limit theorem and also looking at Equation 2.1.6 can suggest that the resulting SPL is due to a random process over frequency. This perception, however, misses to appreciate Kuttruff's (2000) explanations that recognize the sound field in a room is the result of a deterministic process. As such, once the energy equilibrium is reached, the sound pressure level is stationary and could be determined analytically if all the boundary conditions were known with the required accuracy.

These properties of the sound field can be shown in measurements as in Figure 2.1. At low frequencies, below the "Schroeder Frequency", the sound pressure at a given position is defined by few, individual, sparsely overlapping modes (see Figure 2.1e). At much higher frequencies Figure 2.1a shows how the modes overlap and the characteristic Cauchy-Lorentz functions cannot be identified anymore. The corresponding histogram in Figure 2.1b is a graphical representation of the logarithmic Rayleigh distribution in Equation 2.1.6.

A change in location leads to a change in the contributing resonances according to the cosine terms in the numerator of Equation 2.1.3. For the central limit theorem to take effect, independent samples of contributing modes at the different positions are required. This condition is met as trigonometric functions, oscillating with different integer multiples of a fundamental periodicity, are orthogonal to each other (Bronstein, Semendyayev, Musiol, & Mühlig, 2015, 5.3.6.5-2). This holds for arbitrary geometries since the solutions to Dirichlet and Neumann boundary value problems are always orthogonal (Bronstein et al., 2015, 9.1.3.2-4). As a result, it follows that the resonances/standing waves contributing to the sound pressure at two distant positions are uncorrelated to each other and, hence, are independent samples from the same Rayleigh distribution.

Of course, neighboring positions - $\Delta\mathbf{r}$ apart - are correlated to each other, due to continuously defined standing waves $p_i(\mathbf{r})$. A small change in position will not necessarily lead to a sufficient exchange of the contributing resonances and hence the sound pressure will not immediately show the postulated Rayleigh